Multithreading Semantics: A static plural semantics

We present a extension of a standard, compositional single-sentence semantics (say Heim & Kratzer 1998) into a working plural semantics, capturing everything from donkey pronouns to discourse plurals and quantificational subordination. We start with the idea that discourse is comprised of several separate threads, each with its own existentially bound variables introduced by indefinites. Quantifiers store and retrieve new threads.

Single Threads Formally, non-quantified expressions are interpreted, as usual, relative to an assignment g from variables $\{a, \ldots, z\}$ to sets of individuals. Indefinites carry superscript indices introducing new variables, and pronouns carry subscript indices retrieving values, although both act as variables semantically (cf. Heim 1982):

(1) $[\![\text{A}^w \text{ woman waved to a}^f \text{ friend of hers}_w]\!]^g = 1$ iff $g(w)$ is a woman, $g(f)$ is a friend of $g(w)$'s, and $g(w)$ waved to $g(f)$.

Multiple sentences form a thread θ, whose **discourse state** $\|\theta\|$ is a pair $\langle\beta, \gamma\rangle$, where β is the set of bound variables (those introduced by indefinites) in θ and γ is the set of assignments that satisfy θ. These are calculated as in (2), where σ is a single sentence, and $[\![\phi]\!]_\beta$ is the set of bound variables in any expression ϕ.[1] Notice that combining an existing thread θ with a new σ results in the union of the two sets of bound variables β but the intersection of the two sets of satisfying assignments γ. Similarly, $[\![\phi]\!]_\beta$ is calculated as in (3), where indefinites introduce bound variables, which are collected up the tree.

(2) $\|\sigma\| = \left\langle [\![\sigma]\!]_\beta, \{g : [\![\sigma]\!]^g = 1\} \right\rangle$; $\|\theta\ \sigma\| = \|\theta\| + \|\sigma\|$ where $\langle\beta_1, \gamma_1\rangle + \langle\beta_2, \gamma_2\rangle \triangleq \langle\beta_1 \cup \beta_2, \gamma_1 \cap \gamma_2\rangle$

(3) Indef's: $[\![\alpha^x]\!]_\beta = \{x\}$; other term'ls: $[\![\alpha]\!]_\beta = \varnothing$; branching nodes: $[\![[\phi\ \psi]]\!]_\beta = [\![\phi]\!]_\beta \cup [\![\psi]\!]_\beta$

Finally, we introduce the notion of a **restricted state**, useful below: the subset of assignments in a state whose values for free variables all come from a given **free-variable assignment**, as defined in (4) – i.e., the subset of γ whose outputs differ from g at most in their values for variables in β. A thread θ is felicitous and true iff its discourse state is nonempty when restricted to the empty assignment: i.e., when $\|\theta\| \sim \varnothing \neq \varnothing$. This implicitly existentially binds all variables at the thread-level.

(4) $\langle\beta, \gamma\rangle \sim g \triangleq \{h \in \gamma : h[\beta]g\}$ where $h[\beta]g$ iff $dom(h \backslash g) \subseteq \beta$

Multiple Threads Quantifiers store and retrieve new threads via new, uppercase variables $\{A, A', A'', A^3 \ldots, Z, Z' \ldots\}$. These variables appear as superscripts on quantifier arguments, as in (5-a), and subscripts on **discourse plurals**, as in (5-b). Any indefinites introduced within a superscripted phrase ϕ^X will be existentially bound at the thread level, and not contribute a bound variable to the embedding thread: i.e., $[\![\phi^X]\!]_\beta = \varnothing$.

Assignments G from uppercase variables to discourse states will be used as a new parameter to the interpretation function, requiring a slight change to our definition of discourse states, as shown in (6). In most cases, G will be ignored and simply passed unchanged from parent to children nodes, but it will be relevant for the interpretation of discourse plural pronouns as in (7) and quantified phrases as in (8), where $\mathfrak{G}$ is the domain of lowercase assignments (g).

(5) a. **Everyone [who owns an^u umbrella]^X [brought it$_u$ to school]$^{X' \subseteq X}$**

[1] This would be a good place to restrict cataphora, as in dynamic systems, if desired.

b. **They$_{X'.u}$ are on that table**

(6) $\|\sigma\|^G = \left\langle [\![\sigma]\!]_\beta, \{g : [\![\sigma]\!]^{g,G}{=}1\}\right\rangle; \;\; \|\theta\ \sigma\|^G = \|\theta\|^G{+}\|\sigma\|^G$

(7) $[\![pro_{X.y}]\!]^{g,G} = [G(X){\sim}g](y)$ where $\gamma(x) \triangleq \bigcup\{g(x) : g{\in}\gamma\}$

(8) $[\![\phi^X]\!]^{g,G} = [G(X){\sim}g](x)$ iff $G(X) = \langle\{x\}, \mathcal{P}(\mathfrak{G})\rangle{+}\|\phi\|^G$
$[\![\phi^{X\subseteq X'}]\!]^{g,G} = [G(X){\sim}g](x)$ iff $G(X) = G(X'){+}\|\phi\|^G$

A pronoun indexed $X.y$ denotes all individual values for y across those h in $G(X)$ which differ from g at most in their values for the bound variables in $G(X)$. A quantified phrase ϕ^X has the same denotation as a pronoun indexed $X.x$. The value for G is restricted via presuppositions and a multi-thread discourse Δ is true iff $\exists G$ such that $\|\Delta\|^G{\sim}\varnothing \neq \varnothing$.

Examples Take a simple donkey sentence like (5-a), which introduces a thread stored in the X series of variables. (5-a) is associated with a G as in (9) and denotations as in (10):

(9) $G(X) = \left\langle \{x, u\},\ \{g : [\![t_x \text{ owns an}^u \text{ umbrella}]\!]^{g,G}{=}1\}\right\rangle$
$G(X') = G(X) + \left\langle \varnothing, \{g : [\![t_x \text{ brought it}_u \text{ to school}]\!]^{g,G}{=}1\}\right\rangle$

(10) $[\![\text{NP}]\!]^{g,G} = [G(X){\sim}g](x)$ (umbrella owners)
$[\![\text{VP}]\!]^{g,G} = [G(X'){\sim}g](x)$ (subset who brought an umbrella to school)
$[\![\text{Every}]\!]^{g,G} = \lambda xy.|x|{=}|y|;\;\; [\![\text{S}]\!]^{g,G}{=}1$ iff $\big|[G(X){\sim}g](x)\big|{=}\big|[G(X'){\sim}g](x)\big|$

A follow-up sentence could include a discourse plural, such as (5-b) or quantificational subordination, such as **Most [of them]$^{X'}$ [put it$_u$ on that table]$^{X''\subseteq X'}$**.

Since there were no free variables in this example, the values for the restriction and nuclear scope sets did not depend on the local value for g. However, consider the following:

(11) **A^w woman entered. Every [t_f friend of hers$_w$]^F [t_f gave her$_w$ a^p present]$^{F'\subseteq F}$.**

For this discourse to be defined relative to a G, $G(F)$ must store the singleton set of bound variables $\{f\}$ and all g such that $g(f)$ is a friend of $g(w)$. This will include cases where $g(w)$ is not a woman. Since w is free in F, $G(F)$ is restricted by the value for w in the local g. In particular, $[G(F){\sim}g](f)$ will return friends of the woman $g(w)$ discussed at the top level.

Next, $G(F')$ will store the domain of bound variables $\{f, p\}$ and exactly those g's in $G(F)$ where $g(f)$ gave $g(w)$ a present $g(p)$. The satisfaction set for the whole discourse will have assignments g such that $g(w)$ is a tall woman who entered, and where $[G(F){\sim}g](f) = [G(F'){\sim}g](f)$ – in other words, where every friend of $g(w)$ gave $g(w)$ a present. A similar process allows quantifiers to be embedded within other quantifiers: the values of the higher quantifier restrict those retrieved from the lower one.

Discussion Unlike dynamic plural systems, we have separated out a global assignment G for thread states, treated differently from individual assignments. This move allows us to capture two additional phenomena: paycheck pronouns as in (12), and internal discourse plurals (i.e., **them** in (13) is only the signatory countries $X'.x$ not $X.x$, cf. Keshet 2019):

(12) **Jasmine λ_x spent the [paycheck of hers$_x$]^P. Marcus λ_x deposited it$_{P.p}$.**

(13) **Most [North Atlantic countries]^X [have signed a treaty designating an attack on one of them$_{X'.x}$ an attack on all of them$_{X'.x}$]$^{X'\subseteq X}$.**