Multi-threading Semantics

Steven Abney & Ezra Keshet

University of Michigan

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Three steps to a plural semantics

With two small extensions to Heim (1982, Chp 2), we can derive a fully functional plural semantics:

1. Indefinites are variables, bound within a thread defined by quantifiers, negation, or the top-level discourse (Heim, 1982).
2. NEW: Such threads may be stored and used later just like predicates.
3. NEW: Plurals are also formed by collecting values from stored threads.

The denotation of an open formula

- Following Heim (1982), we take pronouns *and* indefinites both to denote variables.

(1) A^f friend of hers$_w$ smiled. $\rightsquigarrow fr(f, w) \wedge sm(f)$

- An open formula is true under an assignment of values to its variables. We write:

(2) $\langle f, w \rangle$ for $\{(\text{“}f\text{”}, f), (\text{“}w\text{”}, w)\}$

- Thinking of an assignment as a kind of tuple, the set of assignments that satisfy an open formula constitutes a kind of relation:

(3) $\|fr(f, w) \wedge sm(f)\| = \{\langle f, w \rangle : fr(f, w) \wedge sm(f)\}$

Combining relations

- The relational equivalent of conjunction is **natural join:**

(4) $\|P \wedge Q\| = \|P\| \bowtie \|Q\|$

- Relations represent **discourse state.**

(5) $\|S_1\ S_2\| = \|S_1\| \bowtie \|S_2\|$

- A discourse state counts as true just in case it is nonempty.

Simple Thread

(6) A^{w} woman_{w} waved to a^{f} friend_{f} of hers_{w}

(7) He_{f} smiled.

$$\begin{aligned}
\|(6)\| &= \|wm(w) \wedge fr(f, w) \wedge wv(w, f)\| \\
&= \{\langle w, f\rangle : wm(w) \wedge fr(f, w) \wedge wv(w, f)\} \\
\|(7)\| &= \{\langle f\rangle : sm(f)\} \\
\|(6)\ (7)\| &= \|(6)\| \bowtie \|(7)\| \\
&= \{\langle w, f\rangle : wm(w) \wedge fr(f, w) \wedge wv(w, f) \wedge sm(f)\}
\end{aligned}$$

Truth can be relative

- When converting a relation to a truth value, some variables may escape closure:

(8) It's not true that a$^{\mathsf{f}}$ friend of hers$_{\mathsf{w}}$ smiled.
$\rightsquigarrow \{\langle \mathsf{f} \rangle : fr(\mathsf{f}, \mathsf{w}) \wedge sm(\mathsf{f})\} = \varnothing$

(9) Every$^{\mathsf{f}}$ friend of hers$_{\mathsf{w}}$ smiled.
$\rightsquigarrow \{\mathsf{f} : fr(\mathsf{f}, \mathsf{w})\} = \{\mathsf{f} : fr(\mathsf{f}, \mathsf{w}) \wedge sm(\mathsf{f})\}$

Add some curry

- We can "curry" a relation to factor it into a contextual assignment and a subrelation with lower arity (including possibly arity 0):

(10)
a. $R := \{\langle w, f\rangle : fr(f, w) \wedge sm(f)\}$
b. $R_w = \{\langle f\rangle : fr(f, w) \wedge sm(f)\}$
c. $R_{wf} = \{\langle\,\rangle : fr(f, w) \wedge sm(f)\}$

- Any of them can be closed. Subrelation + closure = application:

$$
\begin{array}{lllll}
(11) & R(w, f) & \leftrightarrow & R_{wf} \neq \varnothing & \leftrightarrow & fr(f, w) \wedge sm(f) \\
 & R(w) & \leftrightarrow & R_w \neq \varnothing & \leftrightarrow & \exists f[fr(f, w) \wedge sm(f)] \\
 & R() & \leftrightarrow & R \neq \varnothing & \leftrightarrow & \exists f \exists w[fr(f, w) \wedge sm(f)]
\end{array}
$$

Syntactically bound and free

- Meanings are open formulas, hence all variables are free. But some are **syntactically bound:** written as superscripts.

(12) a^{f} friend of hers_w smiled

- Which variables are curried out: the syntactically free ones. For example in **negation** (cf. Heim's selective existential closure):

(13) a. not [a friend^f of hers_{w} smiled]
b. $\|fr(f, w) \wedge sm(f)\|_{\text{w}} = \varnothing$
$\leftrightarrow \neg\exists f[fr(f, w) \wedge sm(f)]$

Set abstraction

- Set abstraction. Extracts a column of the relation.

(14) $R.x = \bigcup_{g \in R} g(x)$

- Also here, curry out free variables:

(15) every^f $[_{\mathrm{NP}}\ t^f$ friend of hers$_w]$ smiled

a. $\|NP\|_w.f = \{f : \|NP\|_w(f)\}$

b. $= \{f : fr(f, w) \wedge sm(f)\}$

Named relations

- Formulas may be named and used later as propositional variables:

(16) Most^f [fr's of hers$_w$]$^{\mathsf{F}}$ [e_F gave her$_w$ a^p present]^G.

a. $F = \{\langle f, w\rangle : fr(f, w)\}$
b. $G \quad = \{\langle f, w, p\rangle : F(f, w) \wedge pr(p) \wedge gv(f, w, p)\}$

Plural Values

(16) Most^f [fr's of hers$_w$]^F [t_{F_w} gave her$_w$ a^p present]^G.

- Collecting values for particular variables in relations gives us all manner of useful plurals:

  (17) a. $F_w.f$ 'w's friends'
  b. $G_w.f$ 'w's friends who gave w a present'
  c. $G_w.p$ 'presents w's friends gave w'

- These can define generalized quantifiers and discourse plurals:

  (18) a. (16) $\rightsquigarrow \textsc{Most}(F_w.f, G_w.f)\}$
  b. They$_{G_w.p}$ are on that table $\rightsquigarrow ot(G_w.p)$

Detail

$$\alpha^{A} \rightsquigarrow \frac{A_F}{A = \|\alpha\|}$$

where F are the syntactically free variables of α

Note: the fraction indicates $\frac{\textit{value}}{\textit{presupposition}}$

Quantifier Example

(19) Every^f [$_{NP}$ friend$_f$ of hers$_w$]^F [$_{VP}$ e_F smiled]^S

$$\text{NP} \rightsquigarrow \frac{F_w}{F = \{\langle f, w\rangle : fr(f, w)\}}$$

$$\text{VP} \rightsquigarrow \frac{S_w}{S = \{\langle f, w\rangle : F(f, w) \wedge sm(f)\}}$$

$$(19) \rightsquigarrow \frac{\text{EVERY}\,(F_w.f, S_w.f)}{\begin{array}{l} F = \{\langle f, w\rangle : fr(f, w)\} \\ S = \{\langle f, w\rangle : fr(f, w) \wedge sm(f)\} \end{array}}$$

Donkey Anaphora / Discourse Plural / Subordination

(20) a. Everyone^x [$_{NP}$ who owns an^u umbrella]^O
[$_{VP}$ e_O brought it$_u$ to school]^S

b. They$_{S.u}$ are on that table.

(21) Most^x e_S [$_{VP}$ will forget it at the end of the day]^F

$$(20) \quad \rightsquigarrow \frac{\textsc{Every}\,(O.x, S.x) \wedge ot(S.u)}{\begin{array}{c} O = \{\langle x, u\rangle : um(u) \wedge ow(x,u)\} \\ S = \{\langle x, u\rangle : O(x,u) \wedge br(x,u)\} \end{array}}$$

$$(21) \quad \rightsquigarrow \frac{\textsc{Most}(S.x, F.x)}{F = \{\langle x, u\rangle : S(x,u) \wedge ft(x,u)\}}$$

$$(20);(21) \rightsquigarrow \frac{\textsc{every}\,(O.x, S.x) \wedge ot(S.u) \wedge \textsc{Most}(S.x, F.x)}{\begin{array}{c} O = \{\langle x, u\rangle : um(u) \wedge ow(x,u)\} \\ S = \{\langle x, u\rangle : O(x,u) \wedge br(x,u)\} \\ F = \{\langle x, u\rangle : S(x,u) \wedge ft(x,u)\} \end{array}}$$

Strong Donkey Pronoun

(22) Everyone^x $[_{NP}$ who owns an^u umbrella$]^O$
$[_{VP}$ e_O left it$_u$ at home$]^H$

$$(22) \rightsquigarrow \frac{\textsc{Every}\,(O.x, H.x)}{\begin{array}{l} O = \{\langle x, u\rangle : um(u) \wedge ow(x,u)\} \\ H = \{\langle x, u\rangle : O(x,u) \wedge lf(x, O_x.u)\}\end{array}}$$

- $O_x.u = \{u : O(x,u)\}$ with x free, i.e., the set containing all of x's umbrellas.
- x is bound one level up, in the definition of H.

Paycheck Pronoun

(23) a. Jasmine^j λ_x spent [$_{DP}$ her$_x^p$ paycheck]^P
b. Marcus^m λ_x deposited it$_{P_x.p}$

$$\text{DP} \rightsquigarrow \frac{P_x}{P = \{\langle p, x\rangle : pc(p,x)\}}$$

$$(23) \rightsquigarrow \frac{\|sp(x, P_x.p)\|(x{=}j) \wedge \|dp(x, P_x.p)\|(x{=}m)}{P = \{\langle p, x\rangle : pc(p,x)\}}$$

Internal Discourse Plural

(24) Most^c $[_{NP}$ North Atlantic countries$]^N$
$[_{VP}$ e_N defend each other$_{D.c}]^D$

$$(24) \rightsquigarrow \frac{\textsc{Most}\,(N.c, D.c)}{\begin{array}{l} N = \{\langle c\rangle : na(c)\} \\ D = \{\langle c\rangle : N(c) \wedge df(c, D.c)\} \end{array}}$$

- Each discourses presupposes there is an assignment G valuing uppercase relation variables as specified in the discourse.
- For instance, in (24), this G must be such that the relation in $G(D)$ is $\{\langle c\rangle : G(N)(c) \wedge df(c, G(D).c)\}$.
- If there's more than one such G, maximal values seem to be used (e.g., $G(D)$ will contain all NATO countries).