A cross-linguistic survey of "parts" of fractions

Issues: This study investigates how fractions are linguistically realized in human languages, on the basis of a wide range of cross-linguistic data. Fractions consist of a numerator and a denominator. It will be shown that the numerator and the denominator can include a covert/overt "part" noun in fractions. We provide an analysis for the syntax and semantics of fractions.

Data: As shown in (1), in German, the denominator (i.e. "three") is followed by the suffix *-tel*, which is a shortened form of *Teil* 'part'.

(1) *Jan hat zwei* ***Drittel*** *des Buches gelesen.* [German]
Jan have two three.part of.the book read
'Jan has read two thirds of the book.'

Fractions including "part" are common cross-linguistically. The distribution of "part" is summarized in (2).

(2) Patterns of fractions with "part" (order irrelevant)

a. **[Numerator PART] [Denominator PART]**:
E.g. Burmese [Sino-Tibetan], Garo [Sino-Tibetan], Xong [Hmong–Mien]

b. **[Numerator PART] [Denominator]**:
E.g. Mokilese [Austronesian], Telugu [Dravidian]

c. **[Numerator] [Denominator PART]**:
E.g. Cantonese [Sino-Tibetan], Catalan [Romance], English [Germanic], German [Germanic], Japanese [Japanese], Mandarin [Sino-Tibetan], Portuguese [Romance], Samoan [Austronesian], Spanish [Romance], Vietnamese [Austro-Asiatic]

What the data shows is that either the numerator, or the denominator, or both can occur with "part" cross-linguistically. For instance, in Garo both the numerator and the denominator occur with the classifier for "part" as in (3).

(3) ***bak****-gittam-ni* ***bak****-gini* [Garo, Burling (1961:53)]
CLS_{part}-three-of CLS_{part}-two
'two thirds'

In Samoan, the numerator appears with "part" as in (4). Here, the particle *e* appears before the numeral. The same particle is also used when post-nominal adjectives modify a noun.

(4) *lua* [***vaega*** *e fitu*] [Samoan, Neffgen (1918: 39)]
two part E seven
'two sevenths'

The last pattern where the "part" element is associated with a denominator is observed in many languages. Fractions in German like (1) belong to this type. In Japanese, *bun* 'part' follows the denominator. *No* is a linking element which typically intervenes between a nominal modifier and the modified noun. The "part" element *bun* thus forms a constituent with the denominator but not with the numerator in (5).

(5) [[*san-****bun***] *no ichi*] *no zyosei* [Japanese, Hayashishita & Ueyama (2012: 597)]
three-part GEN one GEN women
'one third of the women'

Analysis: Based on the cross-linguistic patterns of fractions summarized in (2), this study pursues a uniform analysis in which the numerator and the denominator each combine with a covert/overt "part". The denotation of the covert/overt "part" noun in fractions is given in (6).

(6) Fractional "part": $[\![$ PART $]\!] = \lambda n.\lambda X.[\mu(X) = n]$

The existence of a covert "part" noun can be marked by a classifier in some classifier languages such as Burmese, Garo, Mokilese and Xong. The denotations of the numerator and the denominator in *two thirds* are given in (7). (Following Ionin & Matushansky (2018), we assume that denominators are not ordinals.)

(7) a. $[\![$ [two PART] $]\!] = \lambda X.[\mu(X) = 2]$ b. $[\![$ [three PART] $]\!] = \lambda X.[\mu(X) = 3]$

The semantics of number words have received much attention in the literature (Barwiser & Cooper 1981, Landman 2004, Ionin & Matushansky 2006, Nouwen 2010, Kennedy 2015). In this study, we adopt the property analysis of number words developed by Rothestein (2012, 2013, 2017). The numerical expressions in (1) are of type $\langle e,t \rangle$. The $^{\cap}$ function converts these elements of type $\langle e,t \rangle$ into numerals of type n (see also Chierchia 1984, 1998, Partee, 1986, Scontras 2017). The denotations are given in (8).

(8) a. $[\![$ $^{\cap}$[two PART] $]\!] = 2$ (Number measured by μ) b. $[\![$ $^{\cap}$[three PART] $]\!] = 3$ (Number measured by μ)

We now have two numerical expressions of type n. We propose that the core meaning of fractions comes from the FRAC function defined as in (9).

(9) $[\![$ FRAC $]\!]^c$

$= \lambda n_1.\lambda n_2.\lambda u.\lambda v.\exists S.[\Pi(S)(u) \wedge |S|=n_2 \wedge \exists \mu \in M.[\forall s,s' \in S.[\mu(s) = \mu(s')]] \wedge \exists S' \subseteq S.[v = \sqcup S' \wedge |S'|=n_1]$,

where M is a contextually determined set of measurement functions (cf. I&M (2018)).

The first argument of FRAC functions as a numerator, and the second as a denominator. The denotation of *two thirds of the seats* in English is given in (10).

(10) $[\![$ *two* FRAC [*third* PART-*s*] *of the seats* $]\!]^c$

$= \lambda v.\exists S.[\Pi(S)([\![\textit{the seats}]\!]) \wedge |S|=3 \wedge \exists \mu \in M.[\forall s,s' \in S.[\mu(s)=\mu(s')]] \wedge \exists S' \subseteq S.[v=\sqcup S' \wedge |S'|=2]$

Suppose that there are six seats ({a, b, c, d, e, f}) in the context. The meaning of each part of the denotation in (10) is given below.

(11) a. $\mathbf{\Pi(S)([\![\textit{the seats}]\!]) \wedge |S| = 3}$:
S is a non-overlapping cover of $[\![$*the seats*$]\!]$, and the cardinality of S is 3.

b. $\mathbf{\exists \mu \in M.[\forall s,s' \in S.[\mu(s)=\mu(s')]]}$:
All members of S are equal to each other with respect to the measurement function μ.
(E.g. μ(s)=2, ^{OK}{a, b | c, d | e, f}, $^{\#}${a, b, c | d | e, f})

c. $\mathbf{\exists S' \subseteq S.[v=\sqcup S' \wedge |S'|=2]}$
There is a cover of v such that it is a subset of the cover S and its cardinality is 2.
(e.g. {a, b | c, d} or {c, d | e, f} or {a, b | e, f}).

When the denotation in (10) is existentially closed, the denotation in (10) gives the correct interpretation of *two thirds of the seats* in the current context (i.e. there are four seats).

Under the present analysis, numerators and denominators are numerical expression of type n and the core meaning of fractions stems from the FRAC function. It thus has room for cross-linguistic variation in the constituency of the numerator, the denominator and the main noun, given in (2). It is also worth noting that the current proposal is in line with the analysis where some numerical expressions contain a silent nominal like the silent "number" (Kayne (2005) Zewig (2006)). The results of this study present evidence that fractions contain the overt/covert nominal elements, like other numerical expressions.

Selected references: **Ionin & Matushansky**. 2018. *Cardinals: The syntax and semantics of cardinal-containing expressions*. **Kayne**. 2005. A note on the syntax of quantity in English. **Rothstein**. 2017. *The semantics of counting and measuring*.