This paper concerns **improper scope phenomena**, our name for the much-studied cluster of cases when a pronoun has an antecedent which does not syntactically bind it:

(1) a. A dog entered. It sat. *Cross-sentential anaphora*
b. Every student wrote a paper. They are on my desk. *Summation pronouns*
c. The employee who saved her paycheck was wiser than the one who cashed it. *Paycheck pronouns*, Karttunen (1969); Jacobson (2000)
d. Every farmer who owns a donkey beats it. *Donkey pronouns*, Geach (1962)
e. Every student wrote a paper. Most turned it in. *Subordination*, Sells (1985)

The **E-type** approach to these phenomena follows Evans (1977) in assuming **E-type pronouns** akin to definite descriptions, picking out a unique referent satisfying some salient description, such as *the dog who entered* for the pronoun in (1-a). Exactly what makes a description salient or unique is often not formalized, though, leading to problems much discussed in the literature, which require more and more formal complexity (see Heim, 1990, a.o.). **Dynamic** approaches (Kamp, 1981; Heim, 1983; Groenendijk and Stokhof, 1991) instead propose that antecedents like *a dog* update a **context state** to store a discourse referent (in this case, a particular dog) as the value for variable x. A later pronoun, even one in a different sentence, can retrieve the value of x. In attempting to capture the full range of phenomena mentioned above, though, dynamic semanticists have found it necessary to propose ever more complex context states. Moreover, dynamic systems struggle with some cases that the E-type approach captures easily, such as paycheck pronouns.

We propose instead a new static system, based on predicate logic, with three simple changes: (i) following Heim (1982, Chp 2) we propose that indefinites introduce variables existentially closed by higher operators, (ii) in particular we propose an operator Σ that $\exists$-closes indefinites and returns sums of individuals, and (iii) we allow uppercase variables to store and retrieve subformulas. The resulting, static system, covers the empirical ground of both existing approaches without the complexity required to solve problems in each.

Consider in (2) a standard interpretation of predicates and conjunction in terms of sets of assignments (take $M(P)$ as the interpretation of predicate P in the relevant model M). New, optional brackets around variables mark what we call **local variables**, roughly those introduced by determiners, including indefinites. $L(\phi)$ returns all local variables in ϕ:

(2)
$$[\![x]\!]^g = [\![[x]]\!]^g = g(x) \qquad L(x)=\varnothing,\ L([x])=\{x\}$$
$$[\![P(\tau_1,\tau_2,\ldots)]\!] = \{g : \langle [\![\tau_1]\!]^g, [\![\tau_2]\!]^g, \ldots\rangle \in M(P)\} \qquad L(P(\tau_1,\tau_2,\ldots)) = L(\tau_1)\cup L(\tau_2)\cup\ldots$$
$$[\![\phi\wedge\psi]\!] = [\![\phi]\!]\cap[\![\psi]\!] \qquad L(\phi\wedge\psi) = L(\phi)\cup L(\psi)$$

We next add a **summation operator** $\ulcorner\Sigma_x\phi\urcorner$ (cf. Kamp and Reyle, 1993), defined in (3), which (a) existentially closes x and all local variables in ϕ, and (b) denotes the mereological sum of values for x within ϕ. ($\ulcorner g[x, L(\phi)]h\urcorner$ implements $\exists$-closure by allowing h to vary from g just in its values for x and variables in $L(\phi)$.) Note: the definition of $L(\Sigma_x\phi)$ consumes all local variables. Summing over a discourse's world variable as in (4) returns a proposition, capturing cross-sentential anaphora by $\exists$-closing all local variables at the discourse level:

(3)
$$[\![\Sigma_x\phi]\!]^g = \Sigma\{h(x) : h\in[\![\phi]\!]\ \&\ g[x, L(\phi)]h\} \qquad L(\Sigma_x\phi)=\varnothing$$

(4) A^d dog entered. It$_d$ sat. $\rightsquigarrow \Sigma_w\,(\textsc{dog}([d],w)\wedge\textsc{entered}(d,w)\wedge\textsc{sat}(d,w))$

Last, we introduce a system of uppercase **formula variables** (c.f. Keshet, 2018): superscripts $\ulcorner {}^{X}\phi \urcorner$ mark an antecedent ϕ, and then $\ulcorner X \urcorner$ alone effectively repeats ϕ verbatim. To implement this, we assume another parameter like M on $[\![]\!]$ and L, which we likewise omit since it is unique to each discourse's root expression Δ. The **antecedent function** for Δ is that function A such that $A(X){=}\phi$ iff $\ulcorner {}^{X}\phi \urcorner$ occurs in Δ:

(5) $[\![X]\!] = [\![A(X)]\!]$, $L(X) = L(A(X))$; $[\![{}^{X}\phi]\!] = [\![\phi]\!]$, $L({}^{X}\phi) = L(\phi)$ only if $A(X){=}\phi$

With the system fully in place, we can now straightforwardly analyze the remaining examples in (1) (world variables are omitted for space reasons only):

(6) a. Every^{f} [farmer who owns a^{d} donkey]^{O} beats it$_{d}$
$\rightsquigarrow$ $\textsc{every}\big(\Sigma_f^O(\textsc{farmer}([f]) \wedge \textsc{donkey}([d]) \wedge \textsc{owns}(f,d)),\ \Sigma_f(O \wedge \textsc{beats}(f,d))\big)$

b. Every^{s} student^{S} [wrote a^{p} paper]^{W} ...
$\rightsquigarrow$ $\textsc{every}\big(\Sigma_s^S \textsc{student}(s), \Sigma_s^W(S \wedge \textsc{paper}([p]) \wedge \textsc{wrote}(s,p))\big) \wedge \ldots$

(i) ... They$_{\Sigma_p W}$ are on my desk. $\rightsquigarrow \ldots \textsc{on-desk}(\Sigma_p(W))$

(ii) ... Most$^{s}_{W}$ turned it$_{p}$ in. $\rightsquigarrow \ldots \textsc{Most}\,(\Sigma_s(W), \Sigma_s(W \wedge \textsc{turned-in}(s,p)))$

c. The^{x} employee who saved her$^{p}_{x}$ paycheck^{P} was wiser than the^{x} one who cashed it$_{\Sigma_p P}$. $\rightsquigarrow$ $\textsc{wiser}\begin{pmatrix} \Sigma_x(\textsc{employee}([x]) \wedge \textsc{saved}\big(x, \Sigma_p^P \textsc{paycheck-of}([p],x)\big)), \\ \Sigma_x(\textsc{employee}([x]) \wedge \textsc{cashed}(x, \Sigma_p P)) \end{pmatrix}$

Notes: We assume that the nuclear scope of a quantifier incorporates the restriction via formula variable (e.g., O in (6-a) and S in (6-b)). Similarly, quantificational subordination occurs when a previous nuclear scope formula variable is incorporated in a later restriction (e.g, W in (6-b-ii)). The analysis of *the* in (6-c) is quite sketchy and missing the uniqueness presupposition, among other features.

Our system is easily convertible to straight predicate logic (plus summation terms), by simply (a) replacing formula variables with their antecedents and (b) introducing explicit existential closure of local variables within summation terms, as shown in (7). (The reader is invited to employ this process to better understand the formulas above.)

(7) $$\textsc{every}\big(\Sigma_f^O(\textsc{farmer}([f]) \wedge \textsc{donkey}([d]) \wedge \textsc{owns}(f,d)),\ \Sigma_f(O \wedge \textsc{beats}(f,d))\big)$$
$$\approx \textsc{every}\begin{pmatrix} \Sigma_f \exists f \exists d(\textsc{farmer}(f) \wedge \textsc{donkey}(d) \wedge \textsc{owns}(f,d)), \\ \Sigma_f \exists f \exists d(\textsc{farmer}(f) \wedge \textsc{donkey}(d) \wedge \textsc{owns}(f,d) \wedge \textsc{beats}(f,d)) \end{pmatrix}$$

Finally, pronouns denoting summations solve another problem for dynamic approaches, namely scoping out of (double) negation as in (8). In particular, a formula variable like O allows $\ulcorner \Sigma_c O \urcorner$ to retrieve the car that John owns, even when O's antecedent is embedded under negation. Assuming that singular pronouns like *it* presuppose their value to be (nonempty and) singular, though, the same set up will not work under single negation as in (9): the pronoun's presupposition will contradict the first clause.

(8) It's not like John$_{j}$ doesn't [own a^{c} car]^{O}. It$_{\Sigma_c O}$ is just in the shop!
$\rightsquigarrow$ $w \notin \Sigma_w\big(w \notin \Sigma_w^O(\textsc{john}([j]) \wedge \textsc{car}([c],w) \wedge \textsc{owns}(j,c,w))\big) \wedge \textsc{in-shop}(\Sigma_c O, w)$

(9) #John$_{j}$ doesn't [own a^{c} car]^{O}. It$_{\Sigma_c O}$ is just in the shop!
$\rightsquigarrow$ $w \notin \Sigma_w^O(\textsc{john}([j]) \wedge \textsc{car}([c],w) \wedge \textsc{owns}(j,c,w)) \wedge \textsc{in-shop}(\Sigma_c O, w)$ iff $|\Sigma_c O|{=}1$